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Neutrino physics and the mirror world:

How exact parity symmetry explains the solar neutrino deficit, the atmospheric neutrino anomaly and the LSND experiment

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Abstract

Evidence for $\bar{\nu}_\mu \to \bar{\nu}_e$ oscillations has been reported at LAMPF using the LSND detector. Further evidence for neutrino mixing comes from the solar neutrino deficit and the atmospheric neutrino anomaly. All of these anomalies require new physics. We show that all of these anomalies can be explained if the standard model is enlarged so that an unbroken parity symmetry can be defined. This explanation holds independently of the actual model for neutrino masses. Thus, we argue that parity symmetry is not only a beautiful candidate for a symmetry beyond the standard model, but it can also explain the known neutrino physics anomalies.

I Introduction

Recently, the LSND Collaboration has found evidence for $\bar{\nu}_\mu \to \bar{\nu}_e$ oscillations [1]. If the anomaly in this experiment is interpreted as neutrino oscillations, then they obtain the range of parameters $\Delta m^2 \sim 3 - 0.2\ eV^2$ and $\sin^2 2\theta \sim 3 \times 10^{-2} - 10^{-3}$. If the interpretation of this experiment is correct then it will lead to important ramifications for particle physics and cosmology.

In addition to the direct experimental anomaly discussed above, and the theoretical argument for non-zero neutrino masses from the observed electric charges of the known particles [2], there are two indirect indications that the minimal standard model is incomplete. Firstly, there are the solar neutrino experiments [3]. There are four experiments which we summarise in table 1, below:

Experiment	Measurement	SSM-BP	SSM-TCL
^{37}Cl[SNU]	2.23 ± 0.23	8 ± 1	6.4 ± 1.4
Kamioka $\left(\frac{Observed}{SSM-BP}\right)$	$0.50 \pm 0.04 \pm 0.06$	1 ± 0.14	0.77 ± 0.17
GALLEX[SNU]	$79 \pm 10 \pm 6$	131.5 ± 7	122.5 ± 7
SAGE[SNU]	73^{+18+5}_{-16-7}	”	”

Table 1 Solar neutrino measurements and theoretical expectations within the Standard Solar Model (SSM) of Bahcall and Pinsonneault [4], SSM-BP, and Turck-Chieze and Lopes [5], SSM-TCL.

Note that the theoretical predictions for the flux of solar neutrinos involve a lot of assumptions, and the true theoretical value may be outside these errors. In particular the analysis by Turck-Chieze and Lopes [5] gives theoretical predictions for the experiments which are different to those of Bahcall and Pinsonneault [4] but which still seem to be too high to be consistent with the data (although it has been argued that the data and the theory may be in-agreement if one takes into account all sources of uncertainty [6]). In view of the above, we do not attempt to propose a particular physics solution which will make all of the experiments agree with the theoretical prediction of Bahcall et al [4] (as is sometimes done). If there is a solar neutrino problem and if new particle physics is the solution then any new particle physics which can reduce the number of solar neutrinos by a large fraction (e.g. 1/2) may be the cause of the apparent disagreement of theory with data. One interesting possibility, which we will assume, is that the deficit of solar neutrinos is

due to vacuum neutrino oscillations [7]. In particular, if the electron neutrino is a maximally mixed combination of two states, then the number of neutrinos expected from the sun will be 0.5 that of the standard model (for a large range of parameters). This would be, in our opinion, an adequate "explanation" of the solar neutrino deficit [8].

Another experiment which seems to be in conflict with theory is the atmospheric neutrino experiment [9]. This experiment measures the ratio of ν_μ/ν_e interactions where the neutrinos are presumed to originate from cosmic ray interactions in the atmosphere. These experiments observe a deficit in the ratio of ν_μ/ν_e interactions when the data is compared with theory. We summarise the situation below:

$$\begin{array}{ll} Kamiokande & 0.60 \pm 0.07(stat) \pm 0.05(sys), \\ IMB & 0.55 \pm 0.05(stat) \pm 0.10(sys), \end{array} \tag{1}$$

where the data has been normalized to the theoretically expected ratio [10]. Recently, the Kamiokande group has examined atmospheric neutrino events with higher energy, $\gtrsim 1.3$ GeV [11]. Because of the higher energy, they can search for path length dependence. They show that their data can be fit if the muon neutrino is a mixture of two states with the parameters

$$\Delta m^2 \simeq 10^{-2} eV^2 \text{ and } \sin^2 2\theta \simeq 1. \tag{2}$$

This result, together with the earlier Kamiokande and IMB results strongly suggest that the muon neutrino is, at least approximately, a maximally mixed combination of two states.

If intergenerational mixing is suppressed as it is in the quark sector, and also as suggested by the LSND experiment, then the only way to get vacuum oscillations large enough to explain the solar neutrino and atmospheric neutrino anomalies is if there exist additional light neutral particles. These additional light particles cannot belong to additional generations of the usual form, since these types of neutrinos couple to the Z-boson and are ruled out by LEP experiments. The only remaining possibilities are that the additional light neutrinos are either
a) gauge singlets,
b) are members of exotic $SU(2)_L$ multiplets [12], or
c) are members of multiplets of a gauge symmetry [which is not $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$].

While gauge singlets might exist, we would expect them to be heavy, since their masses are not protected by electroweak symmetry. Similarly, if the additional neutrinos are members of non-chiral multiplets, gauge symmetry does not protect their

masses, and we would expect them to be heavy. Exotic $SU(2)_L$ multiplets may exist, but are probably unlikely since the charged members of the multiplets have to be sufficiently heavy to avoid being detected in the decays of the Z, W gauge bosons. Also, the additional contributions to the oblique radiative corrections, which would be expected to be quite large, would have to cancel in order to reproduce the success of the standard model. This seems unlikely. Thus, the only remaining possibility is that the additional light neutrinos are chiral members of a gauge symmetry not contained in $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$. This additional gauge symmetry must be broken (since if it was unbroken, the additional neutrinos would be massless, and could not mix with the ordinary neutrinos), and the scale of symmetry breaking must be less than or not much greater than the electroweak symmetry breaking scale (since otherwise, the gauge symmetry would not protect the masses of the additional neutrinos). [This observation would rule out simple grand unified models such as $SO(10)$ as candidates, since, although they contain additional neutral neutrino species, $SO(10)$ symmetry breaking to the standard model gauge symmetry must occur at a scale significantly higher than the electroweak scale. The usual left-right symmetric model is also not appropriate since the scale of $SU(2)_R \otimes U(1)_{B-L}$ breaking must also be significantly higher then the electroweak breaking scale.]

In other words, the assumption that intergenerational mixing is small, which is supported by the LSND experiment and the small intergenerational mixing of the quarks, together with the large vacuum oscillations needed to explain the solar neutrino deficit and atmospheric neutrino anomaly, imply the existence of additional neutrino species. Furthermore, theoretical arguments suggest that these additional neutrino species should be chiral members of a gauge symmetry not contained in $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$. This additional gauge symmetry should be broken at a scale less than, or not much greater than the electroweak symmetry breaking scale.

We believe that the most compelling model which contains additional neutrino species which are chiral members of a gauge symmetry not contained in $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$, is the standard model extended to include an exact parity symmetry. It has been known for a long time, but not widely appreciated, that an exact parity symmetry can be defined if the particle content and the gauge symmetry of the standard model is doubled [13]. Not only does exact parity symmetry demand the existence of additional light neutrino states, but more importantly if neutrinos are massive and parity is unbroken, then the weak eigenstate will be maximally mixed combinations of mass eigenstates [14, 15].

This means that for a large allowed range of parameters, the flux of electron neutrinos from the sun will be *predicted* to be half that of the standard model, and the number of atmospheric muon neutrinos will be *predicted* to be half that of

the standard model. *Both of these predictions are in good agreement with the data, whereas the standard model is not.*

The outline of this paper is as follows: In section II we review the exact parity model. In section III we show that if neutrinos have mass and the ordinary and mirror neutrinos mix together, then in general the oscillations will be maximal. We show that this result can explain the solar neutrino deficit, the atmospheric neutrino anomaly, and is consistent with the LSND experiment. In section IV we give a naturalness argument, which indicates that the parameters needed to explain the atmospheric neutrino deficit and the LSND result together imply that the expected range of δm^2 for solar neutrinos is in the correct range to explain the deficiency. In section V we illustrate the results of the previous experiments in a concrete model for neutrino masses. The model we use is the usual see-saw model extended so that it is parity symmetric. In section VI we examine a variant of the exact parity model, the exact C invariant model, which also has a mirror sector. This model has very similar predictions and is in general very similar to the exact parity model. In section VII we discuss the incompatibility of the exact parity model (and exact C model) with the standard big bang model of cosmology. In section VIII we conclude with some comments.

II Exact parity symmetric models

In the important paper on possible parity violation in weak interactions, Lee and Yang [13] not only suggested that parity could be violated in the weak interactions, but also pointed out that parity could be retained by enlarging the particle content to include a mirror sector. Since that time, a number of authors have returned to that idea [17, 18, 19]. In these works (with the exception of Ref. [18] as we shall discuss later), it was thought that the mirror sector could not interact with ordinary matter, and hence was of only cosmological [19] and philosophical interest. Independently of these early works, H. Lew and ourselves [16] realized that parity could be conserved by enlarging the particle content to include a mirror sector. In our paper we wrote down the Lagrangian for that theory, we showed that it was sensible and unbroken parity was a possible vacuum of the Higgs potential. We also observed that the mirror sector could in fact interact with ordinary matter, and hence the idea is testable in the Laboratory.

We now review the exact parity symmetric model [16]. To understand how parity might be conserved, consider a model which successfully describes present experiments. In particular, consider the minimal standard model. This model is described by a Lagrangian $\mathcal{L}_1$. This Lagrangian is not invariant under the usual

parity transformation so it seems parity is violated. However, this Lagrangian may not be complete. If we add to $\mathcal{L}_1$ a new Lagrangian $\mathcal{L}_2$ which is just like $\mathcal{L}_1$ except that all left-handed (right-handed) fermions are replaced by new right-handed (left-handed) fermions which feel new interactions of the same form and strength, then the theory described by $\mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2$ is invariant under a parity symmetry (under this symmetry $\mathcal{L}_1 \leftrightarrow \mathcal{L}_2$). In addition to these Lagrangian terms, there may also be terms which mix ordinary with mirror matter and which are parity invariant. We label this part of the Lagrangian as $\mathcal{L}_{int}$. The terms in $\mathcal{L}_{int}$ are very important since they lead to interactions between ordinary and mirror matter, and hence allow the idea to be experimentally tested in the laboratory.

If we apply the above procedure to the standard model then $\mathcal{L}_1$ is just the standard model Lagrangian. We now add the "mirror matter" as described above, so that the total Lagrangian consists of two parts $\mathcal{L}_1$ and $\mathcal{L}_2$. Then the gauge symmetry of the theory is

$$SU(3)_1 \otimes SU(2)_1 \otimes U(1)_1 \otimes SU(3)_2 \otimes SU(2)_2 \otimes U(1)_2. \tag{3}$$

There are two sets of fermions, the ordinary particles and their mirror images, which transform under the gauge group of Eq.(3) as

$$\begin{array}{ll} f_L \sim (1,2,-1)(1,1,0), & F_R \sim (1,1,0)(1,2,-1), \\ e_R \sim (1,1,-2)(1,1,0), & E_L \sim (1,1,0)(1,1,-2), \\ q_L \sim (3,2,1/3)(1,1,0), & Q_R \sim (1,1,0)(3,2,1/3), \\ u_R \sim (3,1,4/3)(1,1,0), & U_L \sim (1,1,0)(3,1,4/3), \\ d_R \sim (3,1,-2/3)(1,1,0), & D_L \sim (1,1,0)(3,1,-2/3), \end{array} \tag{4}$$

(with generation index suppressed). The Lagrangian is invariant under the discrete Z_2 parity symmetry defined by

$$\begin{array}{c} x \to -x, \ t \to t, \\ G_1^\mu \leftrightarrow G_{2\mu}, \ W_1^\mu \leftrightarrow W_{2\mu}, \ B_1^\mu \leftrightarrow B_{2\mu}, \\ f_L \leftrightarrow \gamma_0 F_R, \ e_R \leftrightarrow \gamma_0 E_L, \ q_L \leftrightarrow \gamma_0 Q_R, \ u_R \leftrightarrow \gamma_0 U_L, \ d_R \leftrightarrow \gamma_0 D_L, \end{array} \tag{5}$$

where $G_1^\mu(G_2^\mu), W_1^\mu(W_2^\mu)$ and $B_1^\mu(B_2^\mu)$ are the gauge bosons of the $SU(3)_1$ $[SU(3)_2]$, $SU(2)_1[SU(2)_2], U(1)_1[U(1)_2]$ gauge forces respectively. The minimal model contains two Higgs doublets which are also parity partners:

$$\phi_1 \sim (1,2,1)(1,1,0), \ \phi_2 \sim (1,1,0)(1,2,1). \tag{6}$$

Note that, although the parity symmetry is not of the standard form, it is theoretically a perfectly reasonable candidate for a parity symmetry. It commutes with

the proper Lorentz group, interchanges x with -x and leads to a theory described by a Lagrangian which treats left and right on an equal footing. Also, by virtue of the CPT theorem, there will be an unbroken T symmetry which also connects the ordinary and mirror particles as partners (so that CPT has the usual form),

An important feature which distinguishes this parity conserving theory from other such theories [20] is that the parity symmetry is assumed to be unbroken by the vacuum. The most general renormalizable Higgs potential can be written in the form

$$V(\phi_1, \phi_2) = \lambda_1(\phi_1^\dagger\phi_1 + \phi_2^\dagger\phi_2 - 2u^2)^2 + \lambda_2(\phi_1^\dagger\phi_1 - \phi_2^\dagger\phi_2)^2, \tag{7}$$

where $\lambda_{1,2}$ and u are arbitrary constants. In the region of parameter space where $\lambda_{1,2} > 0$, $V(\phi_1, \phi_2)$ is non-negative and is minimized by the vacuum

$$\langle\phi_1\rangle = \langle\phi_2\rangle = \begin{pmatrix} 0 \\ u \end{pmatrix}. \tag{8}$$

The vacuum values of both Higgs fields are exactly the same and hence parity is not broken by the vacuum in this theory [21].

If the solar system is dominated by the usual particles, then the theory agrees with present experiments. The idea can be tested in the laboratory because it is possible for the two sectors to interact with each other via $\mathcal{L}_{int}$. In the simplest case that we are considering at the moment (where $\mathcal{L}_1$ is the minimal standard model lagrangian), there are just two possible terms (i.e. gauge invariant and renormalizable) in $\mathcal{L}_{int}$. They are,

(1) The Higgs potential terms $\lambda\phi_1^\dagger\phi_1\phi_2^\dagger\phi_2$ in Eq.(7) and

(2) The gauge boson kinetic mixing term $\mathcal{L}_{mix} = \delta F^1_{\mu\nu}F^{2\mu\nu}$, where $F^i_{\mu\nu} = \partial_\mu B^i_\nu - \partial_\nu B^i_\mu$ $(i = 1, 2)$.

The principal phenomenological effect of the Higgs potential mixing term in (1) is to modify (quite significantly) the interactions of the Higgs boson. This effect will be tested if or when the Higgs scalar is discovered. The details have been discussed in Ref.[14]. The principal phenomenological effect of the kinetic mixing term in (2) is to give small electric charges to the mirror partners of the ordinary charged fermions. This effect has also been discussed previously [16, 18, 22]. The experimental bounds on δ are quite weak (about 10^{-3} from searches for minicharged particles [23]), but a new experiment is underway [24] which will either improve this bound, or discover a minicharged fermion. If this fermion has the same mass as one of the known charged fermions, e.g. the electron mass, then it would give strong experimental support for the parity conserving model. Looking for minicharged

particles and studying the properties of the Higgs boson are the only two ways to experimentally test the minimal parity conserving model. Thus, despite the fact that exact parity symmetry necessarily predicts the existence of new light states, the observed agreement of present experiments (not including the neutrino anomalies) with the standard model can also be viewed as evidence for the parity conserving model. In fact, given that the Higgs particle is heavier than the LEP bound, the only way to discover the new physics predicted by the exact parity model (with existing experiments) is to search for mini-charged fermions which have the same masses as the known charged fermions. The situation changes dramatically if neutrinos have mass [14, 15] as we shall now discuss.

III Neutrino mass and the exact parity symmetric model

If neutrinos are massive, then this will provide an important new way for the mirror world to interact with the known world [14, 15]. This is important since it will allow the idea that parity is a exact symmetry of nature to be put to further experimental test. If neutrinos are massive, then $\mathcal{L}_{int}$ can contain neutrino mass terms which mix the ordinary and mirror matter. (Note that if electric charge is conserved, then it is not possible for $\mathcal{L}_{int}$ to contain mass terms mixing the charged fermions of ordinary matter with mirror matter, however neutrinos may be neutral so such mass terms are possible provided that the neutrinos have masses).

To see the effect of the mixing of ordinary and mirror matter consider the electron neutrino. If there were no mirror matter, then small intergenerational mixing will imply that the weak eigenstate electron neutrino will be approximately a single mass eigenstate, ν_e, with mass m. However if mirror matter exists, then there will be a mirror electron neutrino ν_E. If neutrinos are Majorana states, then the most general mass matrix consistent with parity conservation [Eq.(5)] is

$$\mathcal{L}_{mass} = [\bar{\nu}_{eL}, (\bar{\nu}_{ER})^c] \begin{pmatrix} m & m' \\ m' & m^* \end{pmatrix} \begin{bmatrix} (\nu_{eL})^c \\ \nu_{ER} \end{bmatrix} + H.c. \tag{9}$$

where m' is real (due to parity symmetry). Observe that the parameter m can be taken to be real by a choice of phase for ν_e and ν_E. Diagonalizing this mass matrix, we easily obtain that the weak eigenstates ν_e, ν_E are each maximally mixed combinations of mass eigenstates:

$$\nu_{eL} = \frac{(\nu_1^+ + \nu_1^-)_L}{\sqrt{2}},$$

$$(\nu_{ER})^c = \frac{(\nu_1^+ - \nu_2^-)_L}{\sqrt{2}}, \tag{10}$$

where $\nu_1^+,\ \nu_1^-$ are the mass eigenstates. [Note that the superscripts ($\pm$) refer to the sign under parity transformation: Under parity, $\nu_1^+ \to +(\nu_1^+)^c,\ \nu_1^- \to -(\nu_1^-)^c$.] Thus, the effect of ordinary matter mixing with mirror matter is very dramatic. No matter how small the mass interaction term is, the mixing is maximal. We have shown this here by the specific case of one ordinary neutrino and its mirror. This result is actually more general as we shall see.

This one generation example can easily be extended to three generations. Under the assumption that intergenerational mixing is suppressed, which is after all expected considering what happens in the quark sector, and is supported by the LSND experiment, then the three generation case will be, to a first approximation, 3 copies of the mass matrix Eq.(9). Thus, if nature is described by a parity invariant Lagrangian, and neutrinos are massive, then we would expect each of the three known weak eigenstate neutrinos to be, approximately, maximal mixtures of two physical states. Remarkably, the hypothesis that the electron and the muon neutrinos are maximal mixtures of two physical states solves two of the outstanding neutrino puzzles. The solar neutrino deficit can be explained due to electron neutrino - mirror electron neutrino oscillation. The atmospheric neutrino anomaly can be explained due to muon neutrino - mirror muon neutrino oscillation. The LSND experiment is also accomodated, due to the small mixing between the first and second generations.

What are the parameters needed to account for these phenomena? If we ignore the third generation, then there will be 4 light states, which will be combinations of the 4 weak eigenstates: $\nu_e, \nu_E, \nu_\mu, \nu_M$ (i.e. the electron neutrino and its mirror, the muon neutrino and its mirror). If there were no intergerational mixing, then these 4 weak eigenstates will each be maximally mixed combinations of mass eigenstates of the form:

$$\begin{aligned}
\nu_{eL} &= \frac{(\nu_1^+ + \nu_1^-)_L}{\sqrt{2}}, \\
(\nu_{ER})^c &= \frac{(\nu_1^+ - \nu_1^-)_L}{\sqrt{2}}, \\
\nu_{\mu L} &= \frac{(\nu_2^+ + \nu_2^-)_L}{\sqrt{2}}, \\
(\nu_{MR})^c &= \frac{(\nu_2^+ - \nu_2^-)_L}{\sqrt{2}}.
\end{aligned} \tag{11}$$

Of course, in the real world we expect intergenerational mixing, which means that the weak eigenstates will actually have the form:

$$
\begin{aligned}
\nu_{eL} &= \frac{\cos\theta\nu_{1L}^{+}}{\sqrt{2}} + \frac{\sin\theta\nu_{2L}^{+}}{\sqrt{2}} + \frac{\cos\phi\nu_{1L}^{-}}{\sqrt{2}} + \frac{\sin\phi\nu_{2L}^{-}}{\sqrt{2}},\\
(\nu_{ER})^c &= \frac{\cos\theta\nu_{1L}^{+}}{\sqrt{2}} + \frac{\sin\theta\nu_{2L}^{+}}{\sqrt{2}} - \frac{\cos\phi\nu_{1L}^{-}}{\sqrt{2}} - \frac{\sin\phi\nu_{2L}^{-}}{\sqrt{2}},\\
\nu_{\mu L} &= \frac{-\sin\theta\nu_{1L}^{+}}{\sqrt{2}} + \frac{\cos\theta\nu_{2L}^{+}}{\sqrt{2}} - \frac{\sin\phi\nu_{1L}^{-}}{\sqrt{2}} + \frac{\cos\phi\nu_{2L}^{-}}{\sqrt{2}},\\
(\nu_{MR})^c &= \frac{-\sin\theta\nu_{1L}^{+}}{\sqrt{2}} + \frac{\cos\theta\nu_{2L}^{+}}{\sqrt{2}} + \frac{\sin\phi\nu_{1L}^{-}}{\sqrt{2}} - \frac{\cos\phi\nu_{2L}^{-}}{\sqrt{2}},
\end{aligned}
\tag{12}
$$

where we have assumed that the mass matrix is real. The assumption that the mass matrix is real means that ν_1^+ states can only mix with ν_2^+ states, since a mixing term $m\bar{\nu}_1^+\nu_2^-$ is forbidden by parity invariance if m is real. Similarly the ν_1^- only mix with the ν_2^- states. In general, the weak eigenstates are maximal mixtures of the mass eigenstates states whether the mass matrix is real or complex. This can be understood by observing that the mass eigenstate field must also be a parity eigenstate otherwise parity would be broken. The most general form for the mass eigenstate field, assuming two generations, is:

$$
\psi_L = \alpha\nu_{eL} + \beta(\nu_{ER})^c + \gamma\nu_{\mu L} + \delta(\nu_{MR})^c, \tag{13}
$$

where $|\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1$ Requiring that the mass term $m\bar{\psi}\psi$ [where $\psi \equiv \psi_L + (\psi_L)^c$], be invariant under parity, requires $\alpha = \beta^*$ and $\gamma = \delta^*$ which means that $|\alpha| = |\beta|$ and $|\gamma| = |\delta|$. This means that in general, a mass eigenstate has a 1/2 probability of interacting like ordinary weak eigenstates ν_e or ν_μ and 1/2 probability of interacting like mirror weak eigenstates ν_E or ν_M. This result can easily be extended to any number of generations.

Thus restricting the mass matrix to be real does not matter as far as maximal mixing is concerned. This point was not fully understood in Ref.[15]. In Ref.[15] it was shown that under the assumption that the mass matrix was real, there is maximal mixing of the ordinary and mirror weak eigenstates with respect to the mass eigenstates. However, we have shown here that the reality condition is unnecessary. Maximal mixing of ordinary and mirror matter is completely generic. It is an automatic consequence of the unbroken parity symmetry.

In the particular case of a mass matrix involving only the minimal particle content of three ordinary left-handed fields and their mirror partners, the only phases

which cannot be absorbed into the fields can be moved onto the intergenerational mass mixing terms, and thus, they do not have an important impact on the physics when the intergenerational mixing is small [25]. The assumption of a real mass matrix, has the advantage of simplicity, in that only two parameters θ, ϕ are required to parameterize the intergenerational mixing (in the simple two generation case).

If the four mass eigenstates $(\nu_1^+, \nu_1^-, \nu_2^+, \nu_2^-)$ have masses $m_1^+, m_1^-, m_2^+, m_2^-$, then the parameters required to explain the solar neutrino deficit, atmospheric neutrino anomaly and LSND experiment are:

$$3 \times 10^{-10}\ eV^2 \stackrel{<}{\sim} |\Delta m_1^2| \equiv |m_1^{+2} - m_1^{-2}| \stackrel{<}{\sim}\ 10^{-3}\ eV^2$$

$$|\Delta m_2^2| \equiv |m_2^{+2} - m_2^{-2}| \simeq 10^{-2}\ eV^2$$

$$10^{-1}\ eV^2 \stackrel{<}{\sim} |m_2^{+2} - m_1^{+2}| \stackrel{<}{\sim}\ 3\ eV^2$$

and

$$(\sin 2\theta + \sin 2\phi)^2 \sim 3 \times 10^{-2} - 10^{-3} \tag{14}$$

respectively.

The range for $|m_1^{+2} - m_1^{-2}|$ is obtained by noting that we must average the oscillation probability by taking into account the region of emission of the sun, the region of absorption on Earth, and the energy spectrum of the source. In particular, for the case of 2 state maximal mixing, the averaged oscillation probablity is 1/2, and is applicable for $|\Delta m_1^2| \stackrel{>}{\sim} 3 \times 10^{-10} eV^2$ [26]. The bound $|\Delta m_1^2| \stackrel{<}{\sim} 10^{-3} eV^2$ comes from the atmospheric neutrino anomaly because we must require that the number of electron neutrinos should not be depleted in that experiment. Note that there is a Laboratory bound of $|\Delta m_1^2| \stackrel{<}{\sim} 10^{-2}$ eV^2[27].

For the atmospheric neutrino anomaly, a recent analysis of higher energy events has found a flux dependence on the azimuthal angle [11]. This allows a determination of the mass difference which is $|\Delta m_2^2| \simeq 10^{-2} eV^2$ and the mixing has also been measured to be maximal, or nearly so ($\sin 2\psi \stackrel{>}{\sim} 0.7$). The range of parameters for Δm_1^2 and Δm_2^2 to explain the solar neutrino deficit and atmospheric neutrino anomaly, together imply that $|m_2^{+2} - m_1^{+2}| \simeq |m_2^{-2} - m_1^{-2}|$. This has been used in Eq.(14) to express the range of parameters suggested by LSND as a constraint on $|m_2^{+2} - m_1^{+2}|$ only.

Note that in order to explain the atmospheric neutrino anomaly and the LSND result, it is necessary for $|\Delta m_2^2| \ll m_2^{\pm 2}$ [28]. This is actually not unexpected since this hierarchy is achieved if the mass mixing term $m' \bar{\nu}_{eL} \nu_{ER}$ is much less than the diagonal terms $m \bar{\nu}_{eL} (\nu_{eL})^c + (e \leftrightarrow E)$. This is not unexpected, since it is one way to understand the CKM matrix in the quark sector. The CKM matrix is approximately

diagonal, which can have a natural origin in non-diagonal masses being suppressed relative to the diagonal ones. If this also happens in the lepton sector, not just between different generations, but also between ordinary and mirror neutrinos, then it is natural to expect $m' \ll m$, and hence $|\Delta m_2^2| \ll m_2^{\pm 2}$.

We emphasise that if intergenerational mixing of the neutrinos is suppressed, then the exact parity symmetric model is predictive. It predicts that the electron neutrino oscillates into its effectively sterile mirror partner in a maximal way, so that the flux of solar electron neutrinos will be predicted to be 0.5 that of the minimal standard model (for a large range of Δm_1^2). The exact parity model also predicts that the muon neutrino will oscillate into its effectively sterile mirror partner also in a maximal way. In other words $\sin^2 2\psi = 1$ is another prediction. Both of these predictions are supported experimentally, and will be scrutinized more closely in the near future as more data is taken, and more experiments are done. This is especially true for the atmospheric neutrino anomaly. Experiments will also be able to determine if the muon neutrino oscillates into the tau neutrino or a sterile neutrino (for the parameters of interest for the atmospheric neutrino anomaly). If it is the tau neutrino, then the explanation given by the exact parity model is ruled out. Also, if the mixing is not approximately maximal, the parity model explanation will also be ruled out. It is also very important to realise that the planned SNO and Super-Kamiokande experiments will be able to test whether solar electron neutrinos oscillate into active or sterile species, given that they can detect neutral current processes. Our model of course predicts that these experiments should see a factor of two reduction in both charged *and* neutral current events.

What can we say about the tau neutrino? As in the case of the electron and muon neutrinos, the tau neutrino should also be approximately a maximal mixture of two states. Thus, the exact parity model predicts that the tau neutrino should also oscillate into the effectively sterile mirror neutrino, also in a maximal way. Exact parity symmetry does not impose any restriction on the squared mass difference, so that the oscillation length is not theoretically constrained. However, if the neutrinos follow a hierarchical mass pattern, as the other fermions do, then the tau neutrino will be the heaviest neutrino, and the squared mass difference of the ν_3^+, ν_3^- parity eigenstates will probably be at least as large as the Δm_2^2 for the second generation (which is $10^{-2} eV^2$ according to the atmospheric neutrino anomaly). If this is the case, then it should be possible to experimentally observe the tau neutrino oscillate into its mirror partner. We would also expect small intergenerational mixing between the tau neutrino and the muon and electron neutrinos. This will also be possible to test experimentally. In fact, several existing experiments are currently searching for $\nu_\tau - \nu_\mu$ oscillations. Our model does not give any indication for the tau neutrino

mass. Cosmological arguments suggest that the tau neutrino mass should be less than about 30 eV. This bound comes from demanding that the relic density of tau neutrinos not violate the energy density bound of the universe (note that there may also be an allowed window above about 1 MeV for the tau neutrino mass for which the tau neutrino decays rapidly enough to be within the cosmological bound). If charged leptons are anything to go by, then one might expect $m_{\nu_\tau}/m_{\nu_\mu} \sim m_\tau/m_\mu$, which gives a tau neutrino mass of about 15 eV for a muon neutrino mass of 1 eV. Obviously such a value would put the tau neutrino in the range for a hot dark matter candidate. However, without a predictive scheme for fermion masses it is not possible to draw any firm conclusions.

IV A naturalness argument

Given that $\Delta m_2^2 \approx 10^{-2} eV^2$, and there is intergenerational mixing between the first and second generations as measured by the LSND experiment, then it is possible to calculate, under some simple assumptions, the contribution of Δm_1^2 induced from Δm_2^2 and the intergenerational mixing. We assume that the second generation neutrinos are much heavier than the first generation neutrinos, and the effects of the third generation can be neglected, at least approximately. In the $\nu^\pm$ basis, the mass matrix has the following simple form if the mass matrix is real:

$$\mathcal{M} = \begin{pmatrix} m_1^{'+} & \delta_1 & 0 & 0 \\ \delta_1 & m_2^+ & 0 & 0 \\ 0 & 0 & m_1^{'-} & \delta_2 \\ 0 & 0 & \delta_2 & m_2^- \end{pmatrix}. \tag{15}$$

Diagonalising this mass matrix, assuming the second generation neutrinos are heavier than the first generation neutrinos, i.e. $m_1^\pm \ll m_2^\pm$ we find

$$m_1^+ = m_1^{'+} - \delta_1^2/m_2^+; \ m_1^- = m_1^{'-} - \delta_2^2/m_2^-. \tag{16}$$

Note that there are essentially two contributions to Δm_1^2. A contribution which depends on the $\delta_{1,2}$ parameters (and is also independent of $m_1^{'\pm}$) and there are also terms which depend on $m_1^{'\pm}$. The term which is independent of $m_1^{'\pm}$ is calculable with the parameters identified in Eq.(14). The other terms which depend on $m_1^{'\pm}$ are unknown, however it would be unnatural for there to be a significant cancellation between the contribution we can calculate and the contribution we cannot (unless there is some crazy symmetry). Thus, assuming there is no fine tuning between the

$m_1'^{\pm}$ and δ^2/m_2 contributions, then

$$\Delta m_1^2 \gtrsim \left(\frac{\delta_1^2}{m_2^+}\right)^2 - \left(\frac{\delta_2^2}{m_2^-}\right)^2$$

$$\approx \sin^4\theta\ (\Delta m_2^2) + (\sin^4\theta - \sin^4\phi)\ (m_2^-)^2, \tag{17}$$

where $\sin\theta = \delta_1/m_2^+,\ \sin\phi = \delta_2/m_2^-$ (and we have assumed that $m_1^{\pm} \ll m_2^{\pm}$). The mixing angles $\sin\theta, \sin\phi$ parameterize the intergenerational mixing between the first and second generation, and are identical to the $\sin\theta, \sin\phi$ defined in Eq.(12). Thus, we expect

$$\Delta m_1^2 \gtrsim \sin^4\phi\ (\Delta m_2^2), \tag{18}$$

with $\theta = \phi$. From the LSND experiment, $\sin^2 2\phi > 10^{-3}$, so that we expect

$$\Delta m_1^2 \gtrsim 10^{-9}\ eV^2. \tag{19}$$

Thus, it is interesting that the range of parameters necessary to solve the atmospheric neutrino anomaly, together with the intergenerational mixing as measured by the LSND experiment, imply that the range of parameters for Δm_1^2 is expected to be in the range necessary to reduce the flux of solar electron neutrinos by a factor of 2 (this occurs for $\Delta m_1^2 \gtrsim 10^{-10} eV^2$). If there were no solar neutrino deficit, then the exact parity model would not be able to explain the atmospheric neutrino anomaly and simultaneously account for the LSND experiment in a compelling way. A deficit of solar neutrinos appears to be a necessary consequence.

V The see saw model.

Hitherto, we have discussed neutrino oscillations in the parity symmetric model without focussing on any particular model. This is possible, because parity symmetry allows us to make the prediction that the weak eigenstates will be maximal mixtures of two states, independently of the details of where the masses come from. We now focus briefly on a particular model. As is well known, in the standard model neutrinos must be massless. Thus, if neutrinos have non-zero masses the standard model must be modified. There are very few good ideas for understanding the smallness of the neutrino masses relative to the masses of the other fermions. The simplest possibility known at the moment is the see-saw model [29]. This involves assuming the existence of a gauge singlet right-handed neutrino which develops a large Majorana mass. The see-saw model is a simple way to understand the smallness of the masses of the known (i.e. the three left-handed) neutrinos. In the usual

see-saw model there are two Weyl neutrino fields per generation. Denote these by ν_L and ν_R. The ν_L field is a member of a $SU(2)_L$ doublet while ν_R is a gauge singlet. The usual Higgs doublet can couple the ν_L and ν_R together and its vacuum expectation value will generate a Dirac mass term. Also, since we assume that ν_R is electrically neutral it can have a bare Majorana mass term coupling it to itself. Thus we have two mass terms:

$$\mathcal{L}_{mass} = 2m\bar{\nu}_L\nu_R + M\bar{\nu}_R(\nu_R)^c + H.c. \tag{20}$$

Note that M is a bare mass term, and can take any value, while m is a mass term which is generated when the electroweak gauge symmetry is broken. It is usually assumed that $M \gg m$, since M is not protected by the gauge symmetry. The mass matrix has the form:

$$\mathcal{L}_{mass} = [\bar{\nu}_L, (\bar{\nu}_R)^c] \begin{pmatrix} 0 & m \\ m & M \end{pmatrix} \begin{bmatrix} (\nu_L)^c \\ \nu_R \end{bmatrix} + H.c. \tag{21}$$

Diagonalising this mass matrix yields two Majorana mass eigenstates with masses m^2/M and M (assuming that $M \gg m$). If we denote the mass eigenstates by ν_{light} and ν_{heavy} then they can be written in terms of the weak eigenstates as follows:

$$\begin{aligned} \nu_{lightL} &= \cos\phi \ \nu_L + \sin\phi \ (\nu_R)^c, \\ \nu_{heavyR} &= -\sin\phi \ (\nu_L)^c + \cos\phi \ \nu_R, \end{aligned} \tag{22}$$

where $\tan\phi = m/M$. Thus in the limit $M \gg m$, we see that the light state is essentially ν_L while the heavy state is essentially ν_R.

The see-saw model is a simple extension of the standard model. As in the case of the standard model, it is straightforward to make it exactly parity invariant [15]. In this case, there are four Weyl neutrino fields per generation: ν_L, ν_R and their mirror images N_R, N_L. Under the parity symmetry $\nu_{L,R} \leftrightarrow \gamma_0 N_{R,L}$. Note that since N_R is the parity partner of ν_L it belongs to an $SU(2)_2$ doublet; while N_L being the parity partner of ν_R is a gauge singlet. [Recall that the gauge group is defined in Eq.(3) and the parity transformations are given in Eq.(5).] Assuming the minimal Higgs sector of one ordinary Higgs doublet and its mirror image, then the following mass terms are allowed (where for simplicity we examine only one generation):

$$\begin{aligned} \mathcal{L}_{mass} &= 2m_1(\bar{\nu}_L\nu_R + \bar{N}_R N_L) + 2m_2[\bar{\nu}_L(N_L)^c + \bar{N}_R(\nu_R)^c] \\ &+ M_1[\bar{\nu}_R(\nu_R)^c + \bar{N}_L(N_L)^c] + M_2(\bar{\nu}_R N_L + \bar{N}_L\nu_R) + H.c. \end{aligned} \tag{23}$$

Note that $m_{1,2}$ are mass terms which arise from spontaneous symmetry breaking, while $M_{1,2}$ are bare mass terms. As in the case discussed above, we will assume that $M_{1,2} \gg m_{1,2}$. We will first also assume that the masses are real. This is not an important restriction and it is the only way in which we depart from the most general case. Later we will coment on the general complex case. From Eq.(23) we see that the mass matrix has the form:

$$\mathcal{L}_{mass} = \bar{\nu}_L \mathcal{M} (\nu_L)^c + H.c., \tag{24}$$

where

$$\nu_L = [(\nu_L)^c, N_R, \nu_R, (N_L)^c]^T, \tag{25}$$

and

$$\mathcal{M} = \begin{pmatrix} 0 & 0 & m_1 & m_2 \\ 0 & 0 & m_2 & m_1 \\ m_1 & m_2 & M_1 & M_2 \\ m_2 & m_1 & M_2 & M_1 \end{pmatrix}. \tag{26}$$

The mass matrix can be simplified by changing to the parity diagonal basis $\nu_L^{\pm} = \frac{\nu_L \pm (N_R)^c}{\sqrt{2}}$, and $\nu_R^{\pm} = \frac{\nu_R \pm (N_L)^c}{\sqrt{2}}$. In this basis the mass matrix has the form

$$\mathcal{M} = \begin{pmatrix} 0 & 0 & 0 & m_+ \\ 0 & 0 & m_- & 0 \\ 0 & m_- & M_- & 0 \\ m_+ & 0 & 0 & M_+ \end{pmatrix}, \tag{27}$$

where $m_{\pm} = m_1 \pm m_2$ and $M_{\pm} = M_1 \pm M_2$. The mass matrix can now be easily diagonalised because it is essentially two copies of the 2×2 mass matrix Eq.(21). In the limit $M_{\pm} \gg m_{\pm}$, the mass matrix Eq.(27) has eigenvalues:

$$m_+^2/M_+, m_-^2/M_-, M_+, M_-, \tag{28}$$

and eigenvectors (which are the mass eigenstates):

$$\nu_1 = \nu_L^+,\ \nu_2 = \nu_L^-,\ \nu_3 = \nu_R^+,\ \nu_4 = \nu_R^-. \tag{29}$$

If we had started with Eq.(26) being the most general complex matrix, then Eq.(29) would still follow. To see this, observe that the heavy states with masses $M_{\pm}$, are approximately mixtures of the ν_R and N_L fields, with small admixture of ν_L and N_R fields. Hence, the light states must be approximately comprised of ν_L and N_R fields. Since, by parity invariance, mass eigenstates must also be parity eigenstates (except

in the case where the states are degenerate), it follows that the light states (ν_1, ν_2) must be approximately of the form given in Eq.(29) (since only these combinations are parity eigenstates). Another way of saying this is that the heavy ν_R and N_L fields decouple, leaving the two light neutrino states ν_L and N_R, which must have a mass matrix of the form Eq.(9) by parity invariance. The discussion following Eq.(9) consequently holds for the see-saw model as well as the minimal model without gauge singlet neutrinos.

Thus we conclude that in the one generation case, there is effectively only two state mixing:

$$\begin{aligned} \nu_L &= \frac{\nu_L^+ + \nu_L^-}{\sqrt{2}} \\ &= \frac{\nu_1 + \nu_2}{\sqrt{2}}. \end{aligned} \tag{30}$$

Thus in this case the neutrino oscillation probability averaged over many oscillations is 1/2. In the physical case of three generations, in general ν_L^+ and ν_L^- will each be linear combinations of mass eigenstates. The details depend on the precise form of the mass matrix. However, the assumption that intergenerational mixing is small means that the one generation result will be a good approximation, and that each weak eigenstate (ν_{eL}, $\nu_{\mu L}$, $\nu_{\tau L}$ and their mirror partners) will each be approximately a maximally mixed combination of mass eigenstates.

The see-saw model illustrates the results of the previous sections in a concrete model. Similar results should also occur in any other model for neutrino masses, provided that there are nonzero neutrino-mirror neutrino mass mixing terms.

VI Exact charge conjugation invariance?

One variant of the exact parity model is an exact (unorthodox) charge conjugation invariant model. This is very similar to the exact parity model. The only difference is that the mirror particles are assumed to have the *same* chirality as the ordinary fermions (recall in the exact parity model, the mirror fermions have the opposite chirality to the ordinary fermions). Explicitly, the gauge symmetry of the theory is

$$SU(3)_1 \otimes SU(2)_1 \otimes U(1)_1 \otimes SU(3)_2 \otimes SU(2)_2 \otimes U(1)_2. \tag{31}$$

and the fermions consist of the ordinary particles and their C images, which trans-

form under the gauge group of Eq.(31) as

$$\begin{array}{ll} f_L \sim (1,2,-1)(1,1,0), & F_L \sim (1,1,0)(1,2,-1), \\ e_R \sim (1,1,-2)(1,1,0), & E_R \sim (1,1,0)(1,1,-2), \\ q_L \sim (3,2,1/3)(1,1,0), & Q_L \sim (1,1,0)(3,2,1/3), \\ u_R \sim (3,1,4/3)(1,1,0), & U_R \sim (1,1,0)(3,1,4/3), \\ d_R \sim (3,1,-2/3)(1,1,0), & D_R \sim (1,1,0)(3,1,-2/3), \end{array} \tag{32}$$

(with generation index suppressed). The Lagrangian is invariant under the discrete Z_2 unorthodox C symmetry defined by

$$\begin{array}{c} G_1^\mu \leftrightarrow G_2^\mu,\ W_1^\mu \leftrightarrow W_2^\mu,\ B_1^\mu \leftrightarrow B_2^\mu \\ f_L \leftrightarrow F_L,\ e_R \leftrightarrow E_R, \\ q_L \leftrightarrow Q_L,\ u_R \leftrightarrow U_R\ d_R \leftrightarrow D_R \end{array} \tag{33}$$

As in the exact parity model, there are two Higgs multiplets, ϕ_1 and ϕ_2, which are partners under the discrete symmetry [the Higgs potential is the same as in the exact parity model, see Eq.(7)]. We denote this Z_2 symmetry unorthodox C symmetry, since it is essentially unorthodox parity times ordinary CP [after relabeling $(F_R)^c$ as F_L etc.]. If ordinary CP were conserved, then the exact parity model would also be an exact unorthodox C invariant model. However ordinary CP is violated, so that the exact unorthodox C invariant model is similar but not exactly the same as the exact parity invariant model. Its testible predictions are also similar to the exact parity symmetric model, however they are not exactly the same. In particular, note that in general, even for complex mass matrices, the unorthodox C eigenstates ν^+ and ν^- cannot be coupled together with a mass term. This is because $\bar{\nu}^+\nu^- \to -\bar{\nu}^+\nu^-$ under C transformation. Recall that in the exact parity symmetric model, ν^+ and ν^- could mix together if the mass matrix was complex since under parity, $\bar{\nu}^+\nu^- \to (\bar{\nu^+})^c(\nu^-)^c$. The two cases should be physically distinct if the mass matrix is complex, and this issue should be studied in order to determine precisely how the exact parity and exact C symmetric models could be differentiated experimentally. This issue however, we leave for future work.

VII Conflict with standard big bang nucleosynthesis?

The standard scenario of Big Bang Nucleosynthesis (BBN) can put constraints on the energy density of the universe when it has temperatures of the order of an MeV and below. This in turn can bound the number of relativistic degrees of freedom, which in the standard scenario comprise photons and neutrinos. Over the last few

years, the upper bound on the number of neutrino flavours N_ν has steadily decreased as increasingly more accurate astronomical and nuclear data have become available. Until very recently, these data were consistent with the SM prediction of $N_\nu = 3$. However, a recent analysis [30] argues that the best fit is obtained with $N_\nu = 2$, and furthermore that $N_\nu = 3$ is ruled out at 99.7% C.L. There thus appears to be an incompatibility between the minimal SM of particle physics and the standard Hot Big Bang model of cosmology. Because of the previous apparent success of BBN, it has become standard practice to use compatibility with BBN to put constraints on extensions of the SM of particle physics. The new doubts about BBN also cast doubt on the veracity of this class of bound on new particle physics. It is possible, for instance, that Big Bang cosmology rather than the SM of particle physics will need to be altered because of this conflict [31].

If the mirror matter exists, then it will also be hard to explain the primordial abundances of light elements observed in the universe. This is because one expects three extra neutrino species, as well as the mirror photon to contribute to the energy density of the early universe during the nucleosynthesis era. According to the standard theory, this will cause the universe to expand too rapidly and leads to unacceptable predictions for light element abundances (given the standard assumptions). As discussed above, this is also the case for the SM, but the problem is even more severe in the exact parity model. Just as it is inappropriate to rule out the minimal SM from its incompatibility with BBN, it is also premature to rule out other particle physics models that do not accord with BBN.

Nevertheless, it is interesting to speculate about how BBN and the exact parity model might be reconciled. One might think that this problem could be alleviated if the temperature of the mirror matter is assumed to be much less than the temperature of the ordinary matter. This could be due to some new physics at very high temperatures [32] or, possibly, divine intervention. However, the oscillations of the muon neutrino into the mirror muon neutrino, necessary to solve the atmospheric neutrino anomaly, would put the mirror neutrino in equilibrium with the ordinary matter. This result should follow from the analysis of a sterile neutrino mixing with the muon neutrino. The bound [33]

$$\sin^2 2\theta \; \delta m^2 \lesssim 10^{-6} \; eV^2, \tag{34}$$

which should be obeyed to prevent sterile species from coming into thermal equilibrium due to oscillation, is violated by the parameters necessary to solve the atmospheric neutrino anomaly. The mirror weak interactions should put the mirror muon neutrino into equilibrium with the entire mirror sector. It seems then that either (a) some modification of the usual nucleosynthesis scenario is required, or

(b) some assumptions that underlie the derivation of Eq.(34) need to be examined. Some ideas include:

1) One important assumption behind Eq.(34) is that the neutrino-antineutrino asymmetry is not much larger than the baryon-antibaryon asymmetry. Suppose instead that the initial $\nu - \overline{\nu}$ asymmetry is much larger than the baryon-antibaryon asymmetry (but still small enough to negligibly affect the expansion rate of the universe), for example $\Delta L \equiv \Delta n_\nu / n_\gamma \sim 10^{-5}$. Previous work [33] has shown that for ΔL below a critical value, the dynamical evolution of neutrino and antineutrino number densities reduces ΔL to zero. If on the other hand ΔL is above this critical value, then the initial ΔL can persist [34, 35]. A nonzero ΔL can severely suppress the transition rate from active to sterile neutrinos. For such large values of ΔL the bound Eq.(34) does not apply, and reconciliation between the sterile neutrino solution to the atmospheric neutrino anomaly is possible [35].

2) There could exist a large *negative* cosmological constant at the nucleosynthesis era, which would slow down the expansion rate of the universe at early times, which would have the opposite effect to increasing the number of neutrino species. We do not know if such a possibility has been seriously considered in the literature before, but it seems interesting to us. Of course, the cosmological constant today would have to be much smaller than at the time of nucleosynthesis. Such a time-dependent cosmological constant may be difficult to implement in a natural manner.

3) The non-zero neutrino masses could be due to some new electroweak symmetry breaking mechanism at some quite low scale ($\lesssim$ 1 MeV). This symmetry breaking scale could be associated with a phase transition in the early universe, so that the neutrinos are effectively massless at temperatures above about 3 MeV (i.e. the temperature where the neutrinos go out of thermal equilibrium). If this happens, then in the early universe the oscillations do not occur and the mirror sector does not come into equilibrium with the ordinary sector (at least not through the mechanism of neutrino oscillations). Thus, a temperature difference between the ordinary and mirror worlds could be maintained if it was set up doing very early times due to physics at extremely high energies.

4) It has been shown that if the tau neutrino is very heavy, between 1 and 10 MeV, then the total energy density in the early universe can be doubled without conflict with nucleosynthesis provided that the tau neutrino decays in a particular range of lifetimes and includes electron neutrinos in its decay products [36, 37, 38]. The electron neutrinos in the decay of tau neutrinos convert neutrons into protons, which

has the opposite effect to increasing the energy density in the early universe.

5) It is rather theoretically appealing to have parity symmetry unbroken by the vacuum. However it is worthwhile, in view of the nucleosynthesis difficulties, to mention another possibility. It is possible that the parity symmetry in the exact parity symmetric model is slightly broken [39]. In this case, the large angle neutrino oscillations would no longer be an automatic consequence of the parity symmetry. However, if the symmetry is only broken slightly then large angle neutrino oscillations would still occur for a large range of parameters. Below we illustrate how having parity slightly broken can lead to a model with acceptable nucleosynthesis predictions.

If we assume that the parity symmetry is spontaneously broken in such a way that the mirror photon is massive (i.e. mirror electromagnetism is spontaneously broken) [40], and heavier then the mass of the mirror electron and mirror positron, then this means that the mirror photon will be unstable and rapidly decay into mirror electron-positron pairs. If we also assume that the mirror electron is still the lightest charged mirror fermion, but its mass is changed so that it is slightly heavier that the ordinary electron ($m_e < m_E \stackrel{<}{\sim} 3$ MeV), then the mirror electron-positron pair can annihilate into ordinary electrons and positrons via an intermediate mirror photon [the ordinary electron-positron pair can interact with the mirror photon because of the $U(1)$ kinetic mixing term] [41]. This will be the dominant annihilation channel since we are assuming that the annihilation into mirror photons is not kinematically allowed.

Thus, in the early universe, when the temperature drops to a few MeV, the mirror electrons and positrons will begin to annihilate into the ordinary electrons and positrons. Thus, if they were in equilibrium before this time (which is what we would expect), then the annihilation of the mirror electrons and mirror positrons will heat the ordinary electrons, positrons and ordinary photons (but not the neutrinos, since they have already decoupled). These particles will be hotter than the neutrinos because of this heating. Below the temperature of 1 MeV, the ordinary electrons and positrons have disappeared as well, leaving only the photons and neutrinos (3 ordinary neutrinos as well as 3 mirror neutrinos). Using the usual methodology (i.e. conservation of entropy), we can calculate the temperature of the neutrinos relative to the photons, which is

$$T_\nu = \left(\frac{2}{9}\right)^{\frac{1}{3}} T_\gamma. \tag{35}$$

Note that the temperature of the neutrinos is significantly less than the usual, standard value of $(4/11)^{\frac{1}{3}} T_\gamma$ because of the mirror electron - mirror positron annihilation.

Thus, the energy density of the neutrinos is reduced because of this reheating by a factor

$$\left(\frac{2}{9}\frac{11}{4}\right)^{\frac{4}{3}} = \left(\frac{11}{18}\right)^{\frac{4}{3}} \simeq 0.52 \tag{36}$$

where we have used the fact that the energy density is proportional to the fourth power of the temperature. However, this reduction in energy density is compensated because there are twice as many neutrino species (3 ordinary plus 3 mirror neutrinos). Thus, we end up with an effective energy density of neutrinos which is essentially the same as the standard model case (since $2 \times 0.52 \simeq 1$).

6) Finally, note that the suggestions (1)-(5) all involve working within the standard big bang model of cosmology. It could be that a more radical modification of cosmology is required. One should keep in mind that the standard cosmological model, although simple, and quite successful, also has many open problems. Due to the nature of cosmology, it is difficult to rigorously test the standard theory and it is possible some fundamental changes could be necessary. We should keep an open mind [42].

While the exact parity model has problems explaining the light element abundances within the framework of the standard big bang model, there are other observations which can be viewed as evidence in favour of the exact parity model. There is evidence on both large and small scales that there is additional matter, called dark matter which has so far escaped direct observation. The exact parity model provides a candidate for the dark matter, which is matter comprised of mirror particles [19].

Finally, note that one cosmological advantage of having parity unbroken as opposed to spontaneously broken is that there will be no domain walls. Domain walls tend to be a problem for theories with spontaneously broken symmetries [43], since the energy density of domain walls can overclose the universe.

VIII Conclusions

The exact parity symmetry is a theoretically appealing symmetry beyond the standard model. It is also experimentally appealing, since it predicts the existence of additional light neutrino species, which, due to the parity symmetry, automatically lead to a large angle neutrino oscillations. These oscillations are consistent with the observations of atmospheric neutrinos and the solar neutrino flux.

Thus, the main conclusion is that the atmospheric neutrino anomaly, the solar neutrino deficit and the LSND experiment are all consistent with the predictions/expectations of the standard model extended to include an exact unbroken

parity symmetry. This explanation will be tested more rigorously in the near future as more data is analysed from existing and several new experiments (SNO and Super-Kamiokande for instance). It is a remarkable prospect that exact parity invariance may have to be reconsidered as a serious candidate for an exact symmetry of nature. We eagerly await the experiments!

Note Added

After placing this manuscript on the e-print archive hep-ph, a paper by Z. Berezhiani and R. N. Mohapatra appeared (hep-ph/9505385) which also deals with a mirror matter model in the context of neutrino anomalies, but in a different way.

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[20] There are many examples of models with a exact parity symmetric Lagrangian, which is spontaneously broken by the vacuum. These include models based on $SU(4) \otimes SU(2)_L \otimes SU(2)_R$ [J. C. Pati and A. Salam, Phys. Rev.D10, 275 (1974)], and $SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ [R. N. Mohapatra and J. Pati, Phys. Rev. D11, 566, 2558 (1975); G. Senjanovic and R. N. Mohapatra, Phys. Rev. D12, 1502 (1975); R. Mohapatra and G. Senjanovic, Phys. Rev. D23, 165 (1981)]. These models have parity symmetry interchanging the $SU(2)_L$ gauge bosons with $SU(2)_R$ (along with $x \leftrightarrow -x$ of course). There is no fundamental reason why parity should interchange $SU(2)_L$ with $SU(2)_R$, and there are other parity symmetric models which do not exhibit this feature. These include models based on $SU(3)_c \otimes SU(3)_l \otimes SU(2)_L \otimes U(1)$ [R. Foot and H. Lew, Phys. Rev. D41, 3502 (1990); R. Foot, H. Lew and R. R. Volkas, Mod. Phys. Lett. A8, 1859 (1992)] in which parity symmetry interchanges the $SU(3)_c$ of QCD with a $SU(3)$ of leptonic colour (quark-lepton symmetric models). Another unorthodox parity symmetric model has the parity operation interchanging the $SU(3)$ of QCD with a $SU(3)_L$ which contains the usual weak interactions i.e. the gauge group of the model is $SU(3)_c \otimes SU(3)_L \otimes U(1)$ [R. Foot, Mod. Phys. Lett. A10, 159 (1995).] There are also spontaneously broken parity symmetric models which have a mirror universe. For a model of this form with gauge group, $SU(3) \otimes SU(2) \otimes SU(2) \otimes U(1) \otimes U(1)$ see S. M. Barr, D. Chang, and G. Senjanovic, Phys. Rev. Lett. 67, 2765 (1991). For another model of this type, see R. Foot and H. Lew in Ref. [21]. While there are many different types of spontaneously broken parity symmetric models, there is essentially only one type of unbroken parity symmetric models [modulo trivial extensions, e.g. $SU(5)_{gut} \otimes SU(5)_{gut}$].

[21] In general this Higgs potential can only have two possible vacua; $\langle \phi_1 \rangle = \langle \phi_2 \rangle$, or $\langle \phi_1 \rangle = u, \langle \phi_2 \rangle = 0$. This second vacuum corresponds to the region of parameter

space $\lambda_1 + \lambda_2 > 0,\ \lambda_2 < 0$. The resulting model has been discussed by R. Foot and H. Lew, McGill University, Taiwan institute preprint, hep-ph/9411390 (1994).

[22] B. Holdom (in Ref. [18]) pointed out that a kinetic mixing term could be radiatively induced by assuming the existence of heavy particles which have both ordinary and miror charges. Such a mechanism for generating kinetic mixing is required if the gauge group does not contain $U(1)$ factors (as is the case in grand unified models, for example). Holdom observed that kinetic mixing can cause the mirror particles to couple to the ordinary photon, thus giving them effectively a mini electric charge. Independently of Holdom's work, we also observed this feature of kinetic mixing in the context of a particular model, the parity conserving model (see Ref.[16]). Following R. Foot and X.-G. He, Phys. Lett. B267, 509 (1991) [which seems to be the first place where kinetic mixing terms were proposed as a possible *tree level* term in a gauge theory with two $U(1)$ factors], we observed that the kinetic mixing term can occur at tree level, and should be viewed as a, a priori, fundamental parameter of the theory. There is no need to assume the existence of additional heavy particles.

[23] E. Golowich and R. W. Robinett, Phys. Rev. D35, 391 (1987); S. Davidson, B. Campbell, and D. Bailey, Phys. Rev. D43, 2314 (1991); M. I. Dobroliubov and A. Yu. Ignatiev, Phys. Rev. Lett. 6, 679 (1990).

[24] Science, 267, 1424 (1995).

[25] We may also need to assume that there is no degeneracy among the neutrinos belonging to different generations.

[26] J. N. Bahcall, *Neutrino astrophysics* (Cambridge University Press, Cambridge, 1989).

[27] Particle data group, Phys. Rev. D**50**, 1173 (1994).

[28] Assuming that $m_2^{\pm} > m_1^{\pm}$. If $m_1^{\pm} > m_2^{\pm}$ then Eq.(14) implies that $|\Delta m_1^2| \ll m_1^{\pm 2}$. Thus, either $|\Delta m_2^2| \ll m_2^{\pm 2}$ or $|\Delta m_1^2| \ll m_1^{\pm 2}$ is required.

[29] M. Gell-Mann, P. Ramond, and R. Slansky, in Supergravity, Proceedings of the Workshop, Stony Brook, New York, 1979, edited by P. van Nieuwenhuizen and D. Z. Freedman (North Holland, Amsterdam, 1979); T. Yanagida, Proceedings of the

workshop on unified theories and baryon number in the universe, Tsukuba, Japan, 1979, edited by A. Sawada and A. Sugamoto (KEK Report No. 79-18, Tsukuba-Gun, Ibaraki-Ken, Japan, 1979); R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44, 912 (1980).

[30] N. Hata et al., Ohio State University report No. OSU-TA-6/95, hep-ph/9505319.

[31] It is of course also possible that the SM of particle physics needs modification. One idea is to extend the theory so that massive tau-neutrinos decay rapidly.

[32] See, E. Kolb et al. in Ref. [19] for one such scenario.

[33] A. Dolgov, Sov. J. Nucl. Phys. 33, 700 (1981); R. Barbieri and A. Dolgov, Nucl. Phys. B349, 743 (1991); K. Kainulainen, Phys. Lett. 244B, 191 (1990); K. Enqvist, K. Kainulainen and M. Thomson, Nucl. Phys. B373, 191 (1992); J. Cline, Phys. Rev. Lett. 68, 3137 (1992); X. Shi et. al. Phys. Rev. D48, 2568 (1993).

[34] K. Enqvist, K. Kainulainen and J. Maalampi, Phys. Lett. B244, 186 (1990).

[35] R. Foot and R. R. Volkas, University of Melbourne report UM-P-95/75, hep-ph/9508275, Phys. Rev. Lett. (in press).

[36] A. D. Dolgov and D. P. Kirilova, Int, J. Mod. Phys. A3, 267 (1988); N. Terasawa and K. Sato, Phys. Lett. B185, 412 (1988).

[37] S. Dodelson, G. Gyuk, M. S. Turner, Phys. Rev. Lett. 72, 3754 (1994).

[38] Actually, a heavy tau neutrino could decay quite rapidly in the exact parity symmetric model, since there is no GIM mechanism for the neutrinos, and the decay: $\nu_3^+ \to \nu_3^- + Z^*$ (where Z^* is a virtual Z boson (actually it is the parity diagonal combination $(Z_1 - Z_2)/\sqrt{2}$), and it will decay into $\nu_e, \nu_E, e^{\pm}$, etc.) is not suppressed by any small mixing angle. A rough calculation suggests that there is a small window of m_{ν_τ} between 9 and 10 MeV, where the lifetime is within the bound quoted in Ref [37].

[39] To build a model with parity symmetry slightly broken in the manner described here, it would be necessary to modify the Higgs sector, by adding additional Higgs doublets for example.

[40] A massive mirror photon has quite unusual experimental signatures. See, R. Foot, Mcgill preprint, hep-ph/9407331.

[41] In order for the mirror electron and mirror positrons to annihilate sufficiently so that their relic abundances do not overclose the universe, a lower bound (of about 10^{-5}) on the $U(1)$ kinetic mixing parameter can be calculated.

[42] It is not universally accepted that the standard big bang model is the correct model for the origin of the universe. For an alternative model, see e.g. F. Hoyle, G. Burbidge, and J. V. Narlikar, Astrophys. J. 410, 437 (1993); Astron. Astrophys. 289, 729 (1994). See also, H. Arp et al., Nature 346, 807 (1990).

[43] Ya. B. Zel'dovich, I. Yu. Kobzarev, and L. B. Okun, Sov. Phys. JETP, 40, 1 (1975).